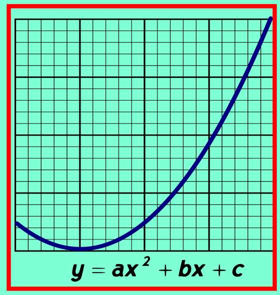
Math 125

Spring 2021

Lecture 14



Class QZ 9

1) Evaluate $\begin{vmatrix} 5 & -2 \\ 3 & 4 \end{vmatrix} = 5(4) - 3(-2) = 20 + 6$

$= \boxed{26}$

2) Solve $x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

using Zero-Factor theorem

$x - 8 = 0$ $\qquad$ $x + 2 = 0$ $\qquad$ $\{-2, 8\}$

$\boxed{x = 8}$ $\qquad$ $\boxed{x = -2}$

Use Cramer's rule to Solve

$$\begin{cases} 3x - 5y = 21 \\ 2x + 3y = -5 \end{cases}$$

$$D = \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = 3(3) - 2(-5) = \boxed{19}$$

$$D_x = \begin{vmatrix} 21 & -5 \\ -5 & 3 \end{vmatrix} = 21(3) - (-5)(-5)$$

$$= 63 - 25$$

$$= \boxed{38}$$

$$D_y = \begin{vmatrix} 3 & 21 \\ 2 & -5 \end{vmatrix}$$

$$= 3(-5) - 2(21)$$

$$= -15 - 42 = \boxed{-57}$$

$$x = \frac{D_x}{D} = \frac{38}{19} = \boxed{2} \qquad y = \frac{D_y}{D} = \frac{-57}{19} = \boxed{-3}$$

$$\{(2, -3)\}$$

Evaluate $\begin{vmatrix} 4 & 0 & -2 \\ 3 & 2 & 0 \\ 0 & 1 & -3 \end{vmatrix}$

$$= 4\begin{vmatrix} 2 & 0 \\ 1 & -3 \end{vmatrix} - 0\begin{vmatrix} 3 & 0 \\ 0 & -3 \end{vmatrix} + (-2)\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= 4(-6-0) - 0(-9-0) - 2(3-0)$$

$$= 4(-6) - 0 - 2(3) = -24 - 6 = \boxed{-30}$$

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Use Cramer's rule to solve for z only.

$$\begin{cases} 2x - 3y + z = 11 \\ x + y - z = -3 \\ x + 2y - 3z = -12 \end{cases} \qquad z = \frac{D_z}{D}$$

Always

$$D = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & -3 \end{vmatrix} = 2\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -1 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 2(-3+2) + 3(-3+1) + 1(2-1)$$

$$= 2(-1) + 3(-2) + 1(1)$$

$$= -2 - 6 + 1 = \boxed{-7}$$

$$D_z = \begin{vmatrix} 2 & -3 & 11 \\ 1 & 1 & -3 \\ 1 & 2 & -12 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 \\ 2 & -12 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -3 \\ 1 & -12 \end{vmatrix} + 11\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 2(-12+6) + 3(-12+3) + 11(2-1)$$

$$= 2(-6) + 3(-9) + 11(1)$$

$$= -12 - 27 + 11 = \boxed{-28}$$

$$z = \frac{D_z}{D} = \frac{-28}{-7}$$

$$\boxed{z = 4}$$

Solving System of linear equations by Matrix Method:

Elementary row operations:

1) You can interchange any two rows.

2) You can multiply or divide any row by any non zero number.

3) You can multiply any row by any non zero number, and add the result to any other row.

These operations will not change the final Answer.

Multiply row 2 by -3

$$\left[\begin{array}{cc|c} 2 & -3 & 5 \\ 1 & 4 & -2 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 2 & -3 & 5 \\ -3 & -12 & 6 \end{array}\right]$$

Divide row 1 by 10

$$\left[\begin{array}{cc|c} 10 & 70 & 120 \\ 2 & -3 & 5 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 7 & 12 \\ 2 & -3 & 5 \end{array}\right]$$

Switch R1 & R3.

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 4 \\ 5 & 1 & 0 & -2 \\ 1 & -3 & 4 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 4 & 0 \\ 5 & 1 & 0 & -2 \\ 2 & 3 & -1 & 4 \end{array}\right]$$

Multiply R1 by -2, and add to R2

$$\left[\begin{array}{cc|c} 1 & 3 & -2 \\ 2 & 1 & 5 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & -5 & 9 \end{array}\right]$$

Multiply R1 by 3, add to R2 ✓

" R1 by -1, add to R3

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 8 \\ -3 & 2 & 0 & 7 \\ 1 & 2 & 3 & 6 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 4 & 3 \\ 0 & 2 & 12 & 31 \\ 0 & 2 & -1 & -2 \end{array}\right]$$

Now $(-1)R2 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 3 \\ 0 & 2 & 12 & 31 \\ 0 & 0 & -13 & -33 \end{array}\right]$$

Matrix Method

$$\left[\begin{array}{cc:c} a & b & c \\ d & e & f \end{array}\right] \Rightarrow \left[\begin{array}{cc:c} 1 & 0 & \# \\ 0 & 1 & \# \end{array}\right]$$

Solutions

Use elementary row operations

$$\begin{cases} x + 2y = -2 \\ 2x + 3y = -2 \end{cases} \Rightarrow \left[\begin{array}{cc:c} 1 & 2 & -2 \\ 2 & 3 & -2 \end{array}\right]$$

Augmented matrix

$(-2)R1 + R2 \rightarrow R2$ $\left[\begin{array}{cc:c} 1 & 2 & -2 \\ 0 & -1 & 2 \end{array}\right]$

$(-1)R2 \rightarrow R2$ $\left[\begin{array}{cc:c} 1 & 2 & -2 \\ 0 & 1 & -2 \end{array}\right]$

$(-2)R2 + R1 \rightarrow R1$ $\left[\begin{array}{cc:c} 1 & 0 & 2 \\ 0 & 1 & -2 \end{array}\right]$ $\Rightarrow \boxed{x=2}$, $\Rightarrow \boxed{y=-2}$

Final Ans

$(2,-2)$

Solve by matrix method:

$$\begin{cases} 3x + y = 7 \\ x - 2y = 0 \end{cases}$$

1) Set-up the augmented Matrix

$$\left[\begin{array}{cc:c} 3 & 1 & 7 \\ 1 & -2 & 0 \end{array}\right]$$

2) Switch R1 & R2.

$$\left[\begin{array}{cc:c} 1 & -2 & 0 \\ 3 & 1 & 7 \end{array}\right]$$

3) $(-3)R1 + R2 \rightarrow R2$

$$\left[\begin{array}{cc:c} 1 & -2 & 0 \\ 0 & 7 & 7 \end{array}\right]$$

4) $R2 \div 7 \rightarrow R2$

$$\left[\begin{array}{cc:c} 1 & -2 & 0 \\ 0 & 1 & 1 \end{array}\right]$$

5) Make -2 to become 0.

$$\left[\begin{array}{cc:c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

$2R2 + R1 \rightarrow R1$

$\boxed{x=2}$

$\boxed{y=1}$

$\boxed{\text{Final Ans: } (2,1)}$

If one row becomes all Zeros
$\Rightarrow$ infinite # of Solutions

$$\left[\begin{array}{ccc:c} 0 & 0 & 0 & 0 \end{array}\right]$$

If one row becomes all Zero except the last number
$\Rightarrow$ NO Solution

$$\left[\begin{array}{ccc:c} 0 & 0 & 0 & \neq 0 \end{array}\right]$$

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Solve by matrix Method

$$\begin{cases} x + y + z = 6 \\ -x + z = 2 \\ y - z = -1 \end{cases}$$

Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ -1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \end{array}\right]$$

$R1 + R2 \rightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 \end{array}\right]$$

$(-)R2 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & -3 & -9 \end{array}\right]$$

$R3 \div (-3) \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$(-2)R3 + R2 \rightarrow R2$, $(-1)R3 + R1 \rightarrow R1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$(-1)R2 + R1 \rightarrow R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} \Rightarrow x = 1 \\ \Rightarrow y = 2 \\ \Rightarrow z = 3 \end{array}$$

Final Ans

$(1, 2, 3)$

Solve by Matrix Method:

$$\begin{cases} x + 2y = 17 \\ 3x + 7y = 47 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 17 \\ 3 & 7 & 47 \end{array}\right]$$

We want to make 3 to become 0.

$(-3)R1 + R2 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & 2 & 17 \\ 0 & 1 & -4 \end{array}\right]$$

We need to make 2 to become 0!

$(-2)R2 + R1 \rightarrow R1$

$$\left[\begin{array}{cc|c} 1 & 0 & 25 \\ 0 & 1 & -4 \end{array}\right] \begin{array}{l} \Rightarrow x = 25 \\ \Rightarrow y = -4 \end{array}$$

Final Ans

$(25, -4)$

Solve by matrix Method: Pivot Point

$$\begin{cases} 2x - 5y - 21z = 39 \\ x - 3y - 10z = 22 \\ x + 3y + 2z = -8 \end{cases}$$

Set-up the Augmented Matrix.

$$\left[\begin{array}{ccc|c} 2 & -5 & -21 & 39 \\ 1 & -3 & -10 & 22 \\ 1 & 3 & 2 & -8 \end{array}\right]$$

Switch R1 & R2, $R1 \leftrightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & -3 & -10 & 22 \\ 2 & -5 & -21 & 39 \\ 1 & 3 & 2 & -8 \end{array}\right]$$

$(-2)R1 + R2 \to R2$

$(-1)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & -3 & -10 & 22 \\ 0 & 1 & -1 & -5 \\ 0 & 6 & 12 & -30 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -10 & 22 \\ 0 & 1 & -1 & -5 \\ 0 & 6 & 12 & -30 \end{array}\right]$$

$R3 \div 6 \to R3$

$$\left[\begin{array}{ccc|c} 1 & -3 & -10 & 22 \\ 0 & 1 & -1 & -5 \\ 0 & 1 & 2 & -5 \end{array}\right]$$

$(-1)R2 + R3 \to R3$

$(3)R2 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -13 & 7 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

$R3 \div 3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -13 & 7 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$(1)R3 + R2 \to R2$

$(13)R3 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \Rightarrow x = 7 \\ \Rightarrow y = -5 \\ \Rightarrow z = 0 \end{array}$$

Final Ans

$(7, -5, 0)$

Solve by matrix method:

$$\begin{cases} x + 2y + 3z = 6 \\ -x - 2y - 3z = -6 \\ 2x + 4y + 6z = 12 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ -1 & -2 & -3 & -6 \\ 2 & 4 & 6 & 12 \end{array}\right]$$

$R1 + R2 \to R2$

$(-2)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

infinite # of Solutions

Solve by matrix method:

$$\begin{cases} x + 2y - 3z = 5 \\ 2x + y + z = 8 \\ 3x + 3y - 2z = 10 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 2 & 1 & 1 & 8 \\ 3 & 3 & -2 & 10 \end{array}\right]$$

$(-2)R1 + R2 \to R2$

$(-3)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & \boxed{-3} & 7 & -2 \\ 0 & -3 & 7 & -5 \end{array}\right]$$

$(-1)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & -3 & 7 & -2 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

All Zeros except

NO Solution

Solve by sub. method:

$$\begin{cases} x^2 + y^2 = 13 \\ y = x+1 \end{cases}$$

$x^2 + (x+1)^2 = 13$

$x^2 + (x+1)(x+1) = 13$

$x^2 + x^2 + x + x + 1 - 13 = 0$

$2x^2 + 2x - 12 = 0$

Divide by 2

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x+3=0 \qquad x-2=0$

$x=-3 \qquad x=2$

$x=2$

$y = 2+1 = 3 \Rightarrow \boxed{(2,3)}$

$x=-3$

$y = -3+1 = -2 \Rightarrow \boxed{(-3,-2)}$

$\{(2,3),(-3,-2)\}$

Class QZ 10

Solve by Cramer's rule

$$\begin{cases} 5x + 3y = 1 \\ 2x - y = 7 \end{cases}$$